Reg. No. | | | | | | | | |

**G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.**

**PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.**
(For those admitted in June 2023 and later)

**PROGRAMME AND BRANCH: M.Sc., MATHEMATICS**

| SEM | CATEGORY | COMPONENT | COURSE CODE | COURSE TITLE |
|---|---|---|---|---|
| II | PART-III | CORE-6 | P23MA206 | PARTIAL DIFFERENTIAL EQUATIONS |

**Date & Session : 28.04.2025/AN Time : 3 hours Maximum: 75 Marks**

| Course Outcome | Bloom's K-level | Q. No. | **SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions. |
|---|---|---|---|
| CO1 | K1 | 1. | The equation $\nabla^2 u + \lambda u = 0$ called as.<br>a) The biharmonic equation<br>b) The Helmholtz equation<br>c) The Poisson equation<br>d) The heat equation |
| CO1 | K2 | 2. | The value $B^2 - 4AC > 0$ is ensure that from the equation $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + F u = G$.<br>a) Hyperbolic Type<br>b) Parabolic Type<br>c) Elliptic Type<br>d) none of these |
| CO2 | K1 | 3. | If the cauchy condition $u = f(\lambda)$ *and* $\frac{\partial u}{\partial n} = g(\lambda)$ on the curve L. where the cauchy data is/are.<br>a) $f(\lambda)$<br>b) $g(\lambda)$<br>c) $f(\lambda)\& g(\lambda)$<br>d) none of these |
| CO2 | K2 | 4. | The characteristic equation of Cauchy problem is.<br>a) $A\left(\frac{\partial y}{\partial x}\right)^2 - B\left(\frac{\partial y}{\partial x}\right) + C = 0$<br>b) $A\left(\frac{\partial y}{\partial x}\right)^2 - B\left(\frac{\partial y}{\partial x}\right) + C \neq 0$<br>c) $A\left(\frac{\partial y}{\partial x}\right)^2 + B\left(\frac{\partial y}{\partial x}\right) + C = 0$<br>d) $A\left(\frac{\partial y}{\partial x}\right)^2 + B\left(\frac{\partial y}{\partial x}\right) + C \neq 0$ |
| CO3 | K1 | 5. | If u may be written as $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$, then the Dirichlet condition make sure that<br>a) u is prescribed in and on boundary<br>b) u is prescribed in a boundary<br>c) u is prescribed on a boundary<br>d) u is prescribed out of the boundary |
| CO3 | K2 | 6. | The equation $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + F u = 0$, then the |

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| | | | equation is elliptic<br>a) A= -C b) A=C c) A=0 d) C=0 |
| CO4 | K1 | 7. | In the Neumann problem $\frac{\partial u}{\partial n} = f(s)$ is harmonic in D, it is also satisfied the following condition.<br>a) $\int f(S)ds = 0$ b) $\int f(S)ds = n$<br>c) $\int f(S)ds \neq 0$ d) $\int f(S)ds \neq n$ |
| CO4 | K2 | 8. | $$u(\rho,\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{1-\rho^2}{1-2\rho\cos(\theta-\tau)+\rho^2} f(\tau)\,d\tau.$$<br>Is called<br>a) poison integral formula for a circle<br>b) poison integral formula for a square<br>c) poison integral formula for a rectangle<br>d) none of these |
| CO5 | K1 | 9. | If the equation<br>$$\nabla^2 u = u_{rr} + \frac{2}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + \frac{\cot\theta}{r^2}u_\theta + \frac{1}{r^2\sin^2\theta}u_{\varphi\varphi},$$<br>where $0 \leq r < a$, $0 < \theta < \pi$, and $0 < \varphi < 2\pi$.<br>Stand for<br>a) Dirichlet Problem for a cube<br>b) Dirichlet Problem for a cylinder<br>c) Dirichlet Problem for a rectangle<br>d) Dirichlet Problem for a Sphere |
| CO5 | K2 | 10. | If the equation<br>$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0,$ for $0 \leq r < a$, $0 < z < l$.<br>Stand for<br>a) Dirichlet Problem for a cube<br>b) Dirichlet Problem for a cylinder<br>c) Dirichlet Problem for a rectangle<br>d) Dirichlet Problem for a Sphere |
| **Course Outcome** | **Bloom's K-level** | **Q. No.** | **SECTION – B (5 X 5 = 25 Marks)**<br>**Answer ALL Questions choosing either (a) or (b)** |
| CO1 | K2 | 11a. | What are assumptions as in the case of the vibrating string to drive the equation for the vibrating membrane?<br>**(OR)** |
| CO1 | K2 | 11b. | To find canonical form of the equation $u_{xx} + u_{xy} + u_{yy} + u_x = 0$. |
| CO2 | K2 | 12a. | To find the solution of the initial boundary-value problem<br>$u_{tt} = 4\,u_{xx}$, $x > 0$, $t > 0$,<br>$u(x, 0) = \|\sin x\|$, $x > 0$,<br>$u_t(x, 0) = 0$, $x \geq 0$,<br>$u(x, 0) = 0$, $t \geq 0$. **(OR)** |
| CO2 | K2 | 12b. | solve<br>$u_{tt} = c^2 u_{xx}$<br>$u(x,0) = \sin x$<br>$u_t(x,0) = \cos x$ |

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| CO3 | K3 | 13a. | Solve<br>$\nabla^2 u = 0 \quad 0 < x < \pi \quad 0 < y < \pi$<br>$u(x,0) = x \quad 0 \le x \le \pi$<br>$u(x,\pi) = 0$<br>$u_x(0,y) = 0$<br>$u_x(\pi,y) = 0$<br>**(OR)** |
| CO3 | K3 | 13b. | State and prove Uniqueness Theorem for the Wave equation. |
| CO4 | K3 | 14a. | Prove that, the solution of the Dirichlet problem, if it exists, is unique.<br>**(OR)** |
| CO4 | K3 | 14b. | The solution of the Dirichlet problem depends continuously on the boundary data. |
| CO5 | K4 | 15a. | Discuss the Dirichlet Problem for a Cube.<br>**(OR)** |
| CO5 | K4 | 15b. | A dielectric sphere of radius a is placed in a uniform electric field E0. Determine the potentials inside and outside the sphere. |

| **Course Outcome** | **Bloom's K-level** | **Q. No** | **SECTION – C (5 X 8 = 40 Marks)**<br>**Answer ALL Questions choosing either (a) or (b)** |
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| CO1 | K4 | 16a. | Derive the two-dimensional wave equation.<br>**(OR)** |
| CO1 | K4 | 16b. | Solve the equation $y^2u_{xx} - x^2u_{yy} = 0$. |
| CO2 | K5 | 17a. | To find the d'Alembert solution of the Cauchy problem for the one-dimensional wave equation.<br>**(OR)** |
| CO2 | K5 | 17b. | solve<br>$u_{tt} = c^2 u_{xx} \quad 0 < x < l \quad t > 0$<br>$u(x,0) = \sin(\pi x / l) \quad 0 \le x \le l$<br>$u_t(x,0) = 0 \quad 0 \le x \le l$<br>$u(0,t) = 0 \quad t \ge 0$<br>$u(l,t) = 0 \quad t \ge 0$ |
| CO3 | K5 | 18a. | Solve<br>$u_{tt} = c^2 u_{xx} \quad 0 < x < l \quad t > 0$<br>$u(x,0) = f(x) \quad 0 \le x \le l$<br>$u_t(x,0) = g(x) \quad 0 \le x \le l$<br>$u(o,t) = 0 \quad t \ge 0$<br>$u(l,t) = 0 \quad t \ge 0$<br>by method of separation.<br>**(OR)** |
| CO3 | K5 | 18b. | Solve |

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| | | | $u_t = ku_{xx} \quad 0 < x < l \quad t > 0$<br>$u(0,t) = 0 \quad t \ge 0$<br>$u(l,t) = 0 \quad t \ge 0$<br>$u(x,0) = x(l-x) \quad 0 \le x \le l$ |
| CO4 | K5 | 19a. | Give the necessary condition for the existence of a solution of the Neumann problem. |
| | | | **(OR)** |
| CO4 | K5 | 19b. | Derive the existence of a solution of the Dirichlet Problem for a Rectangle. |
| CO5 | K6 | 20a. | The necessary condition for the existence of a solution to the Neumann problem for a rectangle. |
| | | | **(OR)** |
| CO5 | K6 | 20b. | To determine the potential in a sphere for Dirichlet Problem for a sphere. |